# SYNOPSIS

**<u>TITLE OF THE RESEARCH</u>: A STUDY ON VARIOUS GENERALIZATION OF FUZZY METRIC SPACES AND DEVELOP FIXED POINT THEORY IN SUCH SPACES**

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After Lotfi A. Zadeh introduced the idea of fuzzy subset in 1965 in order to formulate a theoretical background of Machine intelligence, its demand has enormously been increased day-by-day for its applicability in real life problems involving uncertainty, inconsistency and vagueness.

Generalizing the idea of characteristic function of a subset of crisp set X, Zadeh [10] defined a fuzzy subset A of X as a mapping whose domain is X and range is 1, the closed unit interval [0,1]. For $x \in X$, the value A(x) is called the degree of belongingness (or the grade of membership) of x to the fuzzy subset A.

Fuzzy mathematics has been developed both in pure and applied field viz. fuzzy set theory, fuzzy algebra, fuzzy topology, fuzzy functional analysis, fuzzy dynamical system, fuzzy control, artificial intelligence, pattern recognition etc.

Metric space is an important part in the subject functional analysis. Different authors introduced ideas of fuzzy metric space [1, 2, 7, 8], statistical fuzzy metric space [8], fuzzy pseudo metric space [2] etc. In 1975, Kramosil and Michalek [8], introduced an idea of fuzzy metric on a nonempty set X by a mapping which assigns some grade $\alpha \in (0, 1)$ of distance associated to every pair of points x, y of X and a non-negative real number t. Zike Deng [2], on the other hand, introduced an idea of distance in the set of all fuzzy points of X and defined pseudo metric space

in 1982. In 1984, Kaleva and Sekkala [7] defined fuzzy metric on X as a mapping which associates a fuzzy real number for each pair of elements x, y of X. Combining the idea of Zike Deng and Kaleva and Sekkala, another definition of fuzzy metric was defined by Bandyopadhyay, Samanta and Das [1] in 1989. Using those different ideas studies of fuzzy metric spaces and development of metric fixed point theory have been made by many authors [3, 4, 5, 6]. Recently some authors developed different generalizations of the metric concept in crisp sense. In this direction B. C. Dhage [11] have introduced an idea of D-metric in 1982, which is defined as follows:

Let X be a nonempty set. A function $\rho: X\times X\times X\longrightarrow[0, \infty)$ is called a D- metric on X if

(i) $\rho(x, y, z)=0$ if and only if $x=y=z$ (coincide);

(ii) $\rho(x, y, z)= \rho(p(x, y, z))$ for all $x, y, z \in X$ and for each permutation $p(x, y, z)$ of x, y, z (symmetry);

(iii) ) $\rho(x, y, z)\leq \rho(x, y, a) + \rho(x, a, z)+ \rho(a, y, z)$ for all $x, y, z, a \in$ (tetrahedral inequality).

Some fixed point theorems have been established by different authors [12, 13, 14]. In 2007 S. Sedghi, N. Shobe and H. Zhou [15] have introduced another idea of metric spaces called D*-metric space which is a modification of D-metric space which is as follows :

Let X be a nonempty set. A generalized metric (or D* metric) on X is a function $D^* X\times X\times X\longrightarrow [0, \infty)$ which satisfies the following condition for each $x, y, z, a \in X$:

(i) $D^*(x, y, z) \geq 0$;

(ii) $D^*(x, y, z) = 0$ if and only if $x=y=z$;

(iii) $D^*(x, y, z) = D^*(p(x, y, z))$, (symmetry) where p is a permutation function;

(iv) $D^*(x, y, z) \leq D^*(x, y, a) + D^*(a, z, z)$.

The pair (X, D*) is called a generalized metric (or D*-metric) space.

Authors [15] also established some fixed point theorem in D*-metric space.

**Our plan of work is as follows:**

Firstly we shall study the various types of generalization of fuzzy metric structures such as fuzzy D-metric, fuzzy D*-metric etc. together with their underlying topological properties such as compactness, continuity and contractiveness of mappings. Cauchyness and convergence of sequences and compactness of the spaces will also be studied. Afterwards we shall explore the possibility of developing fuzzy fixed point theory in such spaces. We shall also study on fuzzy cone metric spaces and b-metric spaces.

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